FEDERAL PUBLIC SERVICE COMMISSION
SPECIAL COMPETITIVE EXAMINATION-2023 FOR RECRUITMENT TO POSTS IN BS-17 UNDER THE FEDERAL GOVERNMENT

Roll Number

PURE MATHEMATICS

TIME ALLOWED: THREE HOURS	MAXIMUM MARKS = 100

NOTE: (i) Attempt **FIVE** questions in all by selecting **TWO** Questions each from **SECTION-A&B** and **ONE** Question from **SECTION-C. ALL** questions carry **EQUAL** marks.
(ii) All the parts (if any) of each Question must be attempted at one place instead of at different places.
(iii) Write Q. No. in the Answer Book in accordance with Q. No. in the Q.Paper.
(iv) No Page/Space be left blank between the answers. All the blank pages of Answer Book must be crossed.
(v) Extra attempt of any question or any part of the attempted question will not be considered.
(vi) **Use of Calculator is allowed.**

SECTION-A

Q. No. 1(a) Prove that the homomorphic image φ (G) of a group G is itself a group. **(10)**

(b) Let G be a group and H, K be its subgroups of finite index. Then prove that (H : H ∩ K) = (G : H) if and only if G = HK = KH. **(10)** **(20)**

Q. No. 2(a) Let H be a normal subgroup and K a subgroup of a group G then prove that HK is a subgroup of G, H ∩ K is normal in K and HK/H ≅ K/(H ∩ K). **(10)**

(b) For what value of λ do the following homogeneous equations have nontrivial solutions? Find these solutions: **(10)** **(20)**

$$(3-\lambda)\,x - y + z = 0\,,$$
$$x - (1-\lambda)\,y + z = 0\,,$$
$$x - y + (1-\lambda)\,z = 0\,.$$

Q. No. 3(a) Find a basis and dimension of each of the $R(T)$ and $N(T)$, where $T: R^3 \to R^3$ is defined by **(10)**

$$T\left(x_1, x_2, x_3\right) = \left(x_1 + 2\,x_2 - x_3,\; x_2 + x_3, x_1 + x_2 - 2\,x_3\right).$$

(b) Let U and W be 2-dimensional subspaces of R^3. Show that $U \cap W \neq \{0\}$. **(10)** **(20)**

SECTION-B

Q. No. 4(a) Find constants a and b such that the function f defined by **(10)**

$$f\,(x) = \begin{cases} x^3\,, & if \quad 0 \le x < -1\,, \\ ax + b, & if \quad -1 \le x < 1\,, \\ x^2 + 2, & if \quad x \ge 1\,, \end{cases}$$

is continuous for all x.

(b) Find the extrema of the function $f\,(x, y) = 2\,x^2 + x\,y^2 - 4x - 1\,.$ **(10)** **(20)**

Q. No. 5(a) Evaluate: **(10)**

$$\int_0^{\pi/4} \frac{sec^2\,\theta}{\tan x - \tan\theta}\,dx\,, \qquad \theta > \pi/4\,.$$

(b) Find the area outside of the circle $r = 3$ and inside the cardioid $r = 2\,(1 + \cos\theta)\,.$ **(10)** **(20)**

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Q. No. 6(a) Prove that the straight lines **(10)**

$$\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8} \quad and \quad \frac{x-4}{1} = \frac{y+3}{-4} = \frac{z+1}{7}$$

intersect. Also, find the point of intersection and equation of the plane passing through them.

(b) Discuss and sketch the following curve **(10)** **(20)**

$$y^2 = x^2\left(4 - x^2\right).$$

SECTION-C

Q. No. 7(a) Find the value of **(10)**

$$\oint \frac{\sin\left(\pi z^2\right) + \cos\left(\pi z^2\right)}{(z-1)(z-2)}\, dz$$

around the closed curve which is the circle $|z| = 3$.

(b) Prove that $f(z) = \overline{z}$ is nowhere analytic. **(10)** **(20)**

Q. No. 8(a) Expand $f(z) = \sin z$ in a Taylor series about $z = \pi/4$ and determine the region of convergence of this series. **(10)**

(b) Using residue theorem, evaluate **(10)** **(20)**

$$\frac{1}{2\pi i}\oint \frac{e^{zt}}{z^2\left(z^2 + 2z + 2\right)}\, dz$$

around the closed curve which is the circle $|z| = 3$.

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